D-term inflation with suppressed cosmic strings and lowered n_s

2007 J. Phys. A: Math. Theor. 40 7121

(http://iopscience.iop.org/1751-8121/40/25/S71)

IOP Publishing Journal of Physics A: Mathematical and Theoretical

J. Phys. A: Math. Theor. **40** (2007) 7121–7125 doi:10.1088/1751-8113/40/25/S71

D-term inflation with suppressed cosmic strings and lowered n_s

Osamu Seto[1] **and Jun'ichi Yokoyama**[2]

[1] Department of Physics and Astronomy, University of Sussex, Brighton BN1 9QJ, UK
[2] Research Center for the Early Universe, School of Science, The University of Tokyo, Tokyo 113-0033, Japan

Received 26 October 2006
Published 6 June 2007
Online at stacks.iop.org/JPhysA/40/7121

Abstract

We investigated a simple D-term inflation by taking into account higher order corrections in the potential, particularly, in the Kähler potential. These terms make an inflationary potential flatter than logarithmic in the case without these higher correction terms, through one-loop radiative corrections. As a result, the mass per unit length of cosmic strings formed after inflation can be suppressed and its corresponding $G\mu$ is several $\times 10^{-7}$. In addition, the change of the potential slope simultaneously leads to a more tilted scalar spectral index $n_s \simeq 0.96$–0.97 than that in the model without these corrections $n_s \simeq 0.98$.

PACS numbers: 98.80.Cq, 04.65.+e, 11.27.+d

1. Introduction

Inflation in the early Universe not only realizes globally homogeneous and flat space, but also provides the seeds of density perturbations [1]. To realize successful inflation which matches observational data of large-scale structures and anisotropy of cosmic microwave background radiation (CMB), the potential of the scalar field which drives inflation, *the inflaton*, must be very flat. The required flat potential may be realized with the help of supersymmetry or supergravity. In supersymmetric models, the scalar potential consists of the contribution from F-term and D-term. In F-term inflation models, a slow roll parameter $\eta \equiv V''/V$ generally takes a value of the order of unity, although $\eta \ll 1$ should be satisfied for successful inflation. Here V is the potential energy density of the inflaton; the prime denotes the derivative with respect to the canonical inflaton field and we take the unit with $8\pi G = 1$. This is the so-called η problem of inflation models in supergravity. On the other hand, D-term inflation does not suffer from the problem [2]. Hence, D-term inflation appears more attractive than F-term inflation from this point of view.

However, it has been revealed that the D-term inflation model also has some troubles. First, one may suspect the potential for D-term inflation not to be valid, because the inflaton needs to have a large initial value of the order of (sub-)Planck scale for a natural model parameter [3], although one of the motivations of the hybrid model [4] was inflation within the inflaton field value below the Planck scale [5]. Second, the cosmic strings generated after inflation significantly affect the spectrum of CMB anisotropy [6]. In addition, recently, since the release of WMAP 3 year data which indicate a slightly red tilted spectrum $n_s \sim 0.95$ [7], several researches mind the discrepancy in the scalar spectral index suggested by the observation and predicted by models (see, e.g., [8, 9]), because simple supersymmetric hybrid inflation models predict $0.98 \lesssim n_s \lesssim 1$. Hence, D-term inflation seems to be under strong pressure.

Here, motivated by the first issue mentioned above, we study effects of higher order terms on the Kähler potential in D-term inflation [10], although the leading term in the Kähler potential was taken into account in the study by Rocher and Sakellariadou [11]. These terms alter the dynamics of the inflation and the resultant constraints on the model parameter. As a result, the predicted mass per unit length of cosmic strings can be reduced and meet the observational constraints without assuming a very small Yukawa coupling as in previous works [11, 12]. Moreover, the scalar spectral index also more or less can be reduced.

2. D-term inflation

Here we recall problems by reviewing the D-term inflation [2]. We consider the $\mathcal{N} = 1$ supersymmetric $U(1)$ gauge theory with the non-vanishing Fayet–Iliopoulos (FI) term ξ. The minimal model contains three matter fields S and $\phi_\pm$. The fields $\phi_\pm$ have $U(1)$ charges $q_\pm = \pm 1$ such that $\xi > 0$, while S is neutral for $U(1)$. Supposing the following Kähler potential and superpotential:

$$K = |S|^2 + |\phi_+|^2 + |\phi_-|^2, \qquad W = \lambda S\phi_+\phi_-, \tag{1}$$

the scalar potential is written as

$$V = \lambda^2 \,\mathrm{e}^{|S|^2+|\phi_+|^2+|\phi_-|^2} (|\phi_+\phi_-|^2 + |S\phi_-|^2 + |S\phi_+|^2 + (|S|^2 + |\phi_+|^2 + |\phi_-|^2 + 3)|S\phi_+\phi_-|^2) \\ + \frac{g^2}{2}(\xi + |\phi_+|^2 - |\phi_-|^2)^2, \tag{2}$$

where g is the gauge coupling. The supersymmetric global vacuum is at $(S, \phi_+, |\phi_-|) = (0, 0, \sqrt{\xi})$. For a large value of S,

$$|S| > S_c \equiv \frac{g}{\lambda}\sqrt{\xi}, \tag{3}$$

the potential has the local minimum with a non-vanishing vacuum energy density at $|\phi_\pm| = 0$. The radial part of S is a flat direction but it acquires a non-vanishing potential through radiative corrections, for supersymmetry is broken due to the non-vanishing D-term. In this regime the scalar fields $\phi_\pm$ have masses $m_\pm^2 = \lambda^2|S|^2\,\mathrm{e}^{|S|^2} \pm g^2\xi$, while the mass-squared of their fermionic partner is simply given by $\lambda^2|S|^2\,\mathrm{e}^{|S|^2}$. As a result, the one-loop effective potential is given as

$$V_{\text{1-loop}} = \frac{g^2}{2}\xi^2\left(1 + \frac{g^2}{8\pi^2}\ln\frac{\lambda^2|S|^2\,\mathrm{e}^{|S|^2}}{\Lambda^2}\right), \tag{4}$$

for a large field value $|S|^2 \gg g^2\xi/\lambda$, where Λ is a renormalization scale.

Without loss of generality we can identify the real part of S, $\sigma \equiv \sqrt{2}\,\mathrm{Re}\,S$, as the inflaton. Thus, the inflaton slowly rolls down the potential from a large initial value during inflation. When the inflaton reaches

$$\sigma_c \equiv \sqrt{2}S_c, \tag{5}$$

where ϕ_- becomes tachyonic or

$$\sigma_f \equiv \frac{g}{2\pi}, \tag{6}$$

which corresponds to $\eta(\sigma_f) = -1$, the inflation terminates. Unless the Yukawa coupling λ is extremely small, $\lambda \lesssim 10^{-4}$, inflation terminates when the inflaton arrives at σ_f. In the late stage of inflation, the inflaton evolves as

$$\frac{\sigma^2}{2} - \frac{\sigma_e^2}{2} = \frac{g^2}{4\pi^2}N, \tag{7}$$

where $\sigma_e = \max(\sigma_f, \sigma_c)$ is the field value at the end of inflation and N is the number of e-folds acquired between σ and σ_e. For $N = 50$–60 and a natural value of gauge coupling g, the right-hand side of equation (7) is $\mathcal{O}(0.1)$–$\mathcal{O}(1)$. This means that the inflaton must take a large field value of the order of the sub-Planckian scale.

In the case $\sigma_e = \sigma_f$, by using equation (7), the amplitude of the comoving curvature perturbation is given as

$$\mathcal{P}_\zeta^{1/2} \equiv \frac{H^2}{2\pi|\dot{\sigma}|} = \xi\sqrt{\frac{N}{3}} = 4.7 \times 10^{-5}\left(\frac{\xi}{1.1 \times 10^{-5}}\right)\left(\frac{N}{55}\right)^{1/2}. \tag{8}$$

Thus, the required magnitude of the FI term ξ to generate the appropriate amplitude of the density perturbation is estimated as $\xi = 1.1 \times 10^{-5}$; in other words,

$$\sqrt{\xi} \simeq 3.3 \times 10^{-3} \simeq 7.6 \times 10^{15}\ \text{GeV}. \tag{9}$$

The spectral index is given by $n_s - 1 = -1/N$. Hence, it reads $n_s \simeq 0.98$ for $N \simeq 50$. After inflation, the cosmic string with the mass per unit length $\mu = 2\pi\xi$ is formed. These cosmic strings potentially affect the density perturbation; indeed, this fact is a fatal shortcoming for the D-term inflation model [6]. The constraints on cosmic strings have been studied (for recent studies, see e.g., [12, 14–16]). Referring even a relatively conservative result [12], the constraints on the magnitude of the FI term are derived as

$$\sqrt{\xi} \lesssim 1.9 \times 10^{-3}, \tag{10}$$

and obviously conflict with equation (9).

This problem might be avoided in the other case $\sigma_e = \sigma_c$ with a very small Yukawa coupling, $\lambda \lesssim 10^{-4}$ [12]. This, however, does not entirely solve the problem, because taking a small λ results in a larger value of S_c. If its value exceeds unity, the potential is dominantly lifted by the supergravity effect. As a result, we again need a larger value of ξ to generate the density perturbation with the appropriate magnitude. Indeed, this requires us to adopt not only a small Yukawa coupling λ, but also an anomalously small gauge coupling $g \lesssim 10^{-2}$ [11]. One may find another way to avoid this problem in [9, 13].

3. D-term inflation with higher order couplings

In this section, we take higher order terms in the Kähler potential into account in analyses of the dynamics of D-term inflation. Since we focus on the inflation regime when ϕ_+ and ϕ_- are small, let us consider the following Kähler potential:

$$K = |S|^2 + |\phi_+|^2 + |\phi_-|^2 + f_+(|S|^2)|\phi_+|^2 + f_-(|S|^2)|\phi_-|^2, \tag{11}$$

where $f_\pm(|S|^2)$ are arbitrary functions of $|S|^2$. One can find consequences due to other higher order terms in [10].

Since the condition of the stability for $\phi_\pm$ is unchanged even in the presence of such couplings, we find that inflation would successfully proceed in the same manner as the simple model which we have reviewed in the previous section.

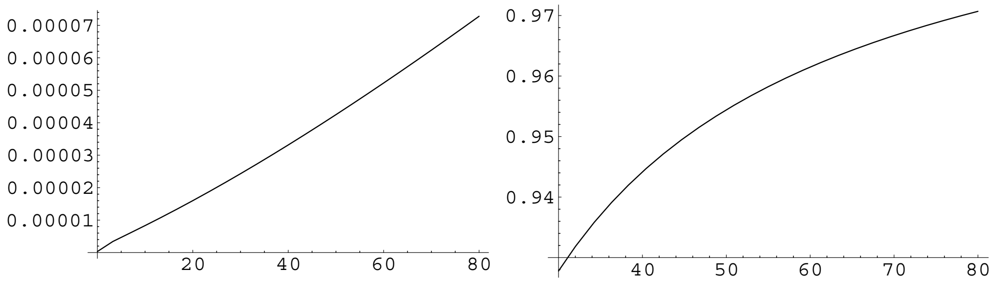

Figure 1. The amplitude of density perturbation $\mathcal{P}_\zeta^{1/2}$ (the left figure) and the scalar spectral index n_s (the right figure) for parameters $g = 0.7$, $c_+ = c_- = 5.5$ and $\xi = 2.7 \times 10^{-6}$. A horizontal axis represents the number of e-folds N and $N \sim 55$ would correspond to the present horizon scale.

Next, we include the radiative correction by $\phi_\pm$ and derive the one-loop effective potential. Here, we should note that $\phi_\pm$ are not canonical any longer owing to the mixing terms in the Kähler potential. The effective masses of charged scalar fields $\phi_\pm$ and their fermionic superpartners are

$$m_{\varphi_\pm}^2 = q_\pm g^2 \xi + \mathrm{e}^{|S|^2} \frac{\lambda^2 |S|^2}{(1+f_+)(1+f_-)} \tag{12}$$

$$m_{\text{fermion}}^2 = \mathrm{e}^{|S|^2} \frac{\lambda^2 |S|^2}{(1+f_+)(1+f_-)} \tag{13}$$

for the canonical variable. The effective potential including the one-loop correction for the canonical inflaton σ is

$$V_{\text{1-loop}}(\sigma) = \frac{g^2\xi^2}{2}\left(1 + \frac{g^2}{8\pi^2}\left[\ln \frac{\lambda^2\sigma^2}{(1+f_+)(1+f_-)\Lambda^2} + \frac{\sigma^2}{2}\right]\right), \tag{14}$$

with $f_\pm = f_\pm(\sigma^2/2)$ for a large field value of σ. The amplitude of the comoving curvature perturbation in this case reads

$$\mathcal{P}_\zeta^{1/2} = \frac{H^2}{2\pi|\dot{\sigma}|} = \frac{3H^3}{2\pi|V'|} = \frac{4\pi\xi}{\sqrt{6}g}\left[\frac{2}{\sigma} + \sigma - \frac{f_{+\sigma}}{1+f_+} - \frac{f_{-\sigma}}{1+f_-}\right]^{-1}, \tag{15}$$

under the slow-roll approximation. Here $f_{\pm\sigma} \equiv df_\pm/d\sigma$. Noting that $f_\pm$ appear in the denominator in equation (14), which means the potential becomes flatter than logarithmic for $f_\pm \gtrsim 1$, we realize, from equation (15), that the Hubble parameter during inflation, equivalently ξ, can be reduced by maintaining the amplitude of the density perturbation $\mathcal{P}_\zeta^{1/2}$.

The followings are the results of an example where we take the lowest order correction to the Kähler potential $f_\pm = c_\pm\sigma^2/2$ with $c_\pm$ being positive constants.

The amplitude of density fluctuation is enhanced due to this flatness of the potential and we can achieve the desired amplitude, $\simeq 10^{-5}$, with smaller values of ξ. From (15) we find that if we take $c = 5.5$ and $g = 0.7$, the amplitude of curvature fluctuation meets the CMB normalization with a small enough value of ξ, $\xi = 2.7 \times 10^{-6}$, as is seen in figure 1, where the spectral index takes $n_s \simeq 0.96$. The magnitude of the FI term is significantly reduced and satisfies equation (10) from the cosmic string constraint. In addition, n_s is also somewhat lowered.

4. Summary

The higher order terms in the Kähler potential can make the potential for the inflaton flatter than logarithmic. The flat potential enables the reduced FI term to accomplish the generation of the appropriate density perturbation and yields the different constraint on the magnitude of the FI term. Then, the influence of the cosmic string formed after inflation on the CMB spectrum can be suppressed. In addition, the change of the potential slope simultaneously leads to a more tilted scalar spectral index $n_s \simeq 0.96$–0.97 than that in the model without these corrections $n_s \simeq 0.98$.

Thus, we conclude that the D-term inflation model can be consistent with the absence of CMB signature from cosmic strings, even if the Yukawa coupling λ is not extremely small. Our model is testable by observations in near future, because the mass per unit length of the cosmic string in our model $G\mu =$ several $\times\, 10^{-7}$ is detectable. In fact, although we have referred to a rather conservative bound on $G\mu$ in this study, if one imposes a less conservative constraint on it, our model is already under strong pressure [16]. In addition, although the model with a very small Yukawa and gauge coupling also predicts the existence of cosmic strings, these models are distinguishable because they predict different spectral indexes.

Acknowledgments

This work was partially supported by PPARC(OS) and by the JSPS grant-in-aid for scientific research no. 16340076 (JY).

References

[1] Guth A H 1981 *Phys. Rev.* D **23** 347
Sato K 1981 *Mon. Not. R. Astron. Soc.* **195** 467
For a review of inflation, see e.g. Linde A D 1990 *Particle Physics and Inflationary Cosmology* (Chur: Harwood)
[2] Halyo E 1996 *Phys. Lett.* B **387** 43
Binetury P and Dvali G R 1996 *Phys. Lett.* B **388** 241
[3] Kolda C and March-Russell J 1999 *Phys. Rev.* D **60** 023504
[4] Linde A D 1994 *Phys. Rev.* D **49** 748
[5] Copeland E J, Liddle A R, Lyth D H, Stewart E D and Wands D 1994 *Phys. Rev.* D **49** 6410
[6] Jeannerot R 1997 *Phys. Rev.* D **56** 6205
[7] Spergel D N *et al* 2006 *Preprint* astro-ph/0603449
[8] Bastero-Gil M, King S F and Shafi Q 2006 *Preprint* hep-ph/0604198
[9] Lin C M and McDonald J 2006 *Phys. Rev.* D **74** 063510
[10] Seto O and Yokoyama J 2006 *Phys. Rev.* D **73** 023508
[11] Rocher J and Sakellariadou M 2005 *Phys. Rev. Lett.* **94** 011303
[12] Endo M, Kawasaki M and Moroi T 2003 *Phys. Lett.* B **569** 73
[13] Kadota K and Yokoyama J 2006 *Phys. Rev.* D **73** 043507
[14] Wyman M, Pogosian L and Wasserman I 2005 *Phys. Rev.* D **72** 023513
[15] Battye R A, Garbrecht B and Moss A 2006 *J. Cosmol. Astropart. Phys.* JCAP09(2006)007
[16] Rocher J and Sakellariadou M 2006 *J. Cosmol. Astropart. Phys.* JCAP11(2006)001 (*Preprint* hep-th/0607226)